Noise of spin-polarized currents at a beamsplitter with local spin-orbit interaction *

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An electronic beamsplitter with a local Rashba spin-orbit coupling can serve as a detector for spin-polarized currents. The spin-orbit coupling plays the role of a tunable spin rotator and can be controlled via a gate electrode on top of the conductor. We use spin-resolved scattering theory to calculate the zero-temperature current fluctuations (shot noise) for such a four-terminal device and show that the shot noise is proportional to the spin-polarization of the source. Moreover, we analyze the effect of spin-orbit induced intersubband coupling, leading to an additional spin rotation.

Shot noise in mesoscopic conductors is widely studied because it yields information about the conductor which is not obtainable by measuring only the average current[1]. These zero-temperature fluctuations can also be used to probe the current *sources*.

Recently, there has been growing interest in sources of *spin-polarized* current[2–4], e.g. for the use in spintronics devices[5]. There is considerable interest not only in producing spin-polarized currents, but also in detecting them in order to assess the functionality of the spin-polarized current sources. Usually, this detection requires spin-selective detection of the current, which is by itself an interesting but difficult task.

In this paper, we describe a method for the detection of spin-polarized currents which relies on the Pauli exclusion principle and does not require spin-sensitive current detection.[6] The proposed setup is shown in Fig. 1. It consists of an electronic beam-splitter[7,8] with a region in one of the ingoing arms (in our case, in lead 1) in which a gate electrode is used to generate and tune the Rashba spin-orbit effect *locally*.[9–12] The current whose spin-polarization is to be determined is injected into both ingoing arms (1 and 2). Electrons injected from leads 1 and 2 are transmitted through the beamsplitter ($1 \to 4$ and $2 \to 3$) with probability $T = |t|^2$ and reflected ($1 \to 3$ and $2 \to 4$) with probability $R = |r|^2 = 1 - T$. We will assume a perfect beamsplitter without backscattering ($1 \to 1$, $2 \to 2$, $1 \to 2$, and $2 \to 1$). Note that the same setup with[6] and without[13] the local Rashba effect has been proposed previously for detecting also spin-entangled electrons.

We use a spin-resolved version of the standard scattering (Landauer-Büttiker) theory[1] and write the total current in lead $\gamma = 1, .., 4$ as

$$I_\gamma(t) = \frac{e}{h}\sum_{\alpha\beta}\int d\varepsilon d\varepsilon' e^{i(\varepsilon-\varepsilon')t/\hbar}\mathbf{a}^\dagger_\alpha(\varepsilon)\mathbf{A}_{\alpha\beta}(\gamma;\varepsilon,\varepsilon')\mathbf{a}_\beta(\varepsilon'),$$

$$\mathbf{A}_{\alpha\beta}(\gamma;\varepsilon,\varepsilon') = \delta_{\gamma\alpha}\delta_{\gamma\beta}\mathbf{1} - \mathbf{s}^\dagger_{\gamma\alpha}(\varepsilon)\mathbf{s}_{\gamma\beta}(\varepsilon'), \qquad (1)$$

where $\mathbf{a}^\dagger_\alpha = (a^\dagger_{\alpha\uparrow}, a^\dagger_{\alpha\downarrow})$, and $a^\dagger_{\alpha\sigma}$ creates an electron with spin σ in lead α. The *spin-dependent* scattering matrix is denoted by $\mathbf{s}$.

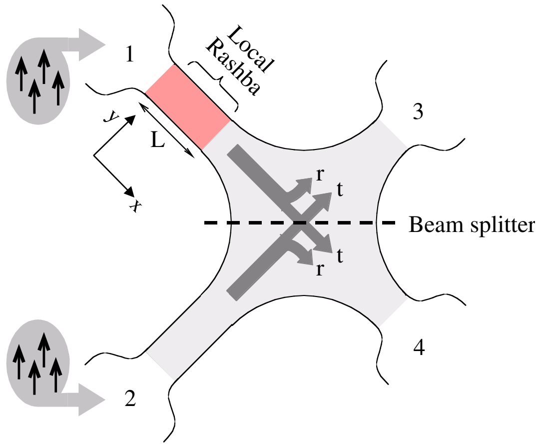

FIG. 1: Beam-splitter geometry in which spin-polarized electrons are injected into leads 1 and 2. The tunable local Rashba spin-orbit interaction in lead 1 can be used to measure the spin-polarization of the incoming current via the current fluctuations (shot noise) measured in one of the outgoing leads (3 or 4). In the case of spin-polarized injection, the shot noise oscillates as a function of the Rashba phase θ_R, while no such oscillations are expected for unpolarized injection. (Adapted from Ref. 6.)

The symmetrized correlator between the current fluctuations $\delta I_\alpha(t) = I_\alpha(t) - \langle I_\alpha\rangle$ in lead α and those in lead β is defined as

$$S_{\alpha\beta}(\omega) = \frac{1}{2}\int dt\,\langle \delta I_\alpha(t)\delta I_\beta(t') + \delta I_\beta(t')\delta I_\alpha(t)\rangle e^{i\omega t}, \quad (2)$$

where $\langle\cdots\rangle = \mathrm{Tr}(\cdots\rho)$ and ρ is the density matrix of the leads in thermal equilibrium. For $\omega = 0$ we obtain

$$S_{\alpha\beta} = \frac{e^2}{h}\mathrm{Re}\int d\varepsilon \sum_{\gamma\delta\sigma\sigma'} A^{\sigma\sigma'}_{\gamma\delta}(\alpha,\varepsilon,\varepsilon)A^{\sigma'\sigma}_{\delta\gamma}(\beta,\varepsilon,\varepsilon) \times f_{\gamma\sigma}(\varepsilon)(1 - f_{\delta\sigma'}(\varepsilon)), \quad (3)$$

where we have used the distribution function $f_{\alpha\sigma}(\varepsilon) = \langle a^\dagger_{\alpha\sigma}a_{\alpha\sigma}\rangle$. Equation (3) is a spin-dependent generalization of Büttiker's formula.[1]

The Rashba spin-orbit coupling[14,15] in a 2D electron system is given by the Hamiltonian

$$H_R = i\alpha(\sigma_y\partial_x - \sigma_x\partial_y), \qquad (4)$$

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where α denotes the spin-orbit coupling constant. In a 1D channel along the x direction (Fig. 1) this Hamiltonian reduces to $H_R = -\alpha k \sigma_y$ and results in a splitting of the two-fold degenerate parabolic conduction band into two spin-orbit subbands as shown in Fig. 2(a). Note that we do not consider a Rashba interaction in lead 2; we are interested only in *phase differences* between leads 1 and 2. However, it is straightforward to extend our analysis to include spin-orbit interaction in both incoming leads.

The effect of a local Rashba spin-orbit region on an electron propagating in x direction (see Fig. 1) can be described by the scattering matrix

$$\mathbf{R} = \begin{pmatrix} \cos\theta_R/2 & -\sin\theta_R/2 \\ \sin\theta_R/2 & \cos\theta_R/2 \end{pmatrix}, \qquad (5)$$

with the Rashba angle $\theta_R = 2m^*\alpha L/\hbar^2$, where m^*, α, and L are the effective mass, spin-orbit constant, and length of the Rashba region. For a typical setup we estimate $\theta_R = \pi$ for $L = 69\,\mathrm{nm}$.[6] Note that a locally applied strong magnetic field in y direction would have a similar effect on the spin. The spin-dependent total scattering matrix $\mathbf{s}_{\alpha\beta}$ is then obtained by multiplying the beamsplitter and Rashba scattering matrices, $\mathbf{s}_{31} = r\mathbf{R}$, $\mathbf{s}_{41} = t\mathbf{R}$, and $\mathbf{s}_{32} = t\mathbf{1}$, $\mathbf{s}_{42} = r\mathbf{1}$. Assuming a perfect beamsplitter without backscattering, we set $\mathbf{s}_{\alpha\alpha} = \mathbf{s}_{12} = \mathbf{s}_{34} = \mathbf{0}$. Substituting $\mathbf{s}_{\alpha\beta}$ into Eq. (3), we obtain for the noise power (current autocorrelator) in one of the outgoing leads (e.g. in lead $\alpha = 3$),

$$S_{33} = \frac{e^2}{h}\int d\varepsilon \Big[(1-T)^2 f_1(\varepsilon)\,(1-f_1(\varepsilon)) + T^2 f_2(\varepsilon)\,(1-f_2(\varepsilon)) + f_3(\varepsilon)\,(1-f_3(\varepsilon)) + T\,(1-T)\sum_{\sigma\sigma'} |R_{\sigma\sigma'}|^2 \{ f_{1\sigma}(\varepsilon)(1-f_{2\sigma'}(\varepsilon)) + f_{2\sigma}(\varepsilon)(1-f_{1\sigma'}(\varepsilon)) \} \Big], \qquad (6)$$

where $f_\alpha = \sum_\sigma f_{\alpha\sigma}$. The first three terms in Eq. (6) describe equilibrium (Johnson-Nyquist) noise originating from thermal fluctuations in lead 1,2, and 3, respectively; these contributions vanish at zero temperature. The remaining two terms describe zero temperature quantum fluctuations (shot noise) which will be our interest here.

From now on we assume zero temperature, thus $f_{\alpha\sigma}(\varepsilon) = \Theta(\mu_{\alpha\sigma} - \varepsilon)$, where Θ denotes the Heaviside step function and $\mu_{\alpha\sigma}$ the electrochemical potential. We model the injection of (partially) spin-polarized currents into the leads 1 and 2 by setting

$$\mu_{1\uparrow} = \mu_{2\uparrow} = \varepsilon_F + eV, \qquad (7)$$

$$\mu_{1\downarrow} = \mu_{2\downarrow} = \varepsilon_F + \frac{1-p}{1+p}eV, \qquad (8)$$

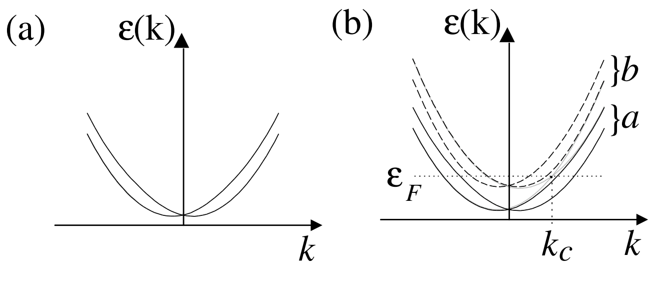

FIG. 2: Conduction band dispersion in the presence of Rashba spin-orbit coupling Eq. (4) for (a) a strictly one-dimensional conductor with a single band, and (b) for two transverse subbands a and b where the Fermi energy ε_F is close to the energy $\varepsilon(k_c)$ corresponding to the avoided subband crossing at $k = k_c$.

$$\mu_{3\sigma} = \mu_{4\sigma} = \varepsilon_F, \qquad (\sigma = \uparrow, \downarrow), \qquad (9)$$

where ε_F denotes the Fermi energy and $0 \le p \le 1$ is the spin polarization of the incoming current. Substituting Eqs. (7)–(9) into Eq. (6), we obtain

$$S_{33} = \frac{2e^2}{h} T(1-T)\,eV\,\frac{2p}{1+p}|R_{\downarrow\uparrow}(\theta_R)|^2. \qquad (10)$$

Dividing this expression by the average current

$$I_3 = \frac{e^2}{h}V\frac{2}{1+p}, \qquad (11)$$

we obtain the Fano factor (noise-to-current ratio)

$$F \equiv \frac{S_{33}}{2eI_3} = T(1-T)\,p\,|R_{\downarrow\uparrow}(\theta_R)|^2, \qquad (12)$$

which is proportional to the polarization p of the incoming current. The fact that the shot noise is completely suppressed for $p = 0$ is due to the Pauli exclusion principle which reduces the number of possible outgoing states. Partial suppression for unpolarized currents was observed experimentally.[7,8] The same explanation holds for polarized electrons (arbitrary p) without spin rotation ($\theta_R = 0$). We assume T to be constant and only consider the reduced Fano factor $f_p \equiv F/T(1-T)$. Using Eq. (5), we obtain

$$f_p = p\,\sin^2(\theta_R/2). \qquad (13)$$

This result is plotted in Fig. 3(a); it shows the possibility of distinguishing spin-polarized from unpolarized

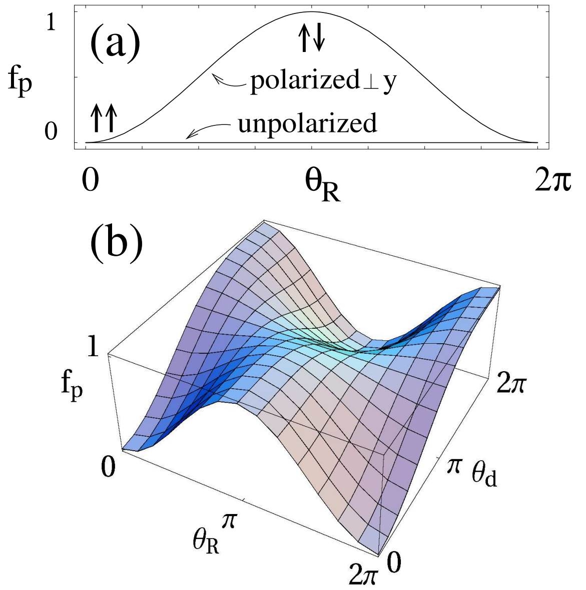

FIG. 3: Reduced Fano factor (noise-to-current ratio) $f_p = F/T(1-T)$ at one of the outgoing leads (3 or 4 in Fig. 1) for electrons polarized perpendicular to the y axis. Figure (a) shows f_p in the single-band case, Fig. 2(a), as a function of the Rashba angle θ_R, see Eq. (13). For unpolarized electrons or electrons polarized in the y direction, we expect $f_p \equiv 0$. At $\theta_R = \pi$, the spins are rotated from a parallel to an antiparallel configuration, giving rise to maximal shot noise. (b) illustrates the two-band case with intersubband coupling, cf. Fig. 2(b). Here, f_p also depends on the intersubband angle θ_d, see Eq. (16).

currents via noise measurements. Note that the spin-polarization must not be collinear to the axis of the local spin rotator (here, the y axis), otherwise the rotation has no effect. Equation (13) could alternatively be used to determine the Rashba coupling strength $\alpha = \hbar^2\theta_R/2m^*L$ via noise measurements.

So far, we have considered a strictly 1D lead [single-band, see Fig. 2(a)] with a local Rashba interaction. We now extend our analysis to the case in which *two* transverse channels a and b are populated, see Fig. 2(b).[16] The intersubband coupling due to the Rashba spin-orbit coupling Eq. (4) is most relevant if the Fermi energy is close to the energy of the avoided crossing of the two transverse subbands, $\varepsilon_F \approx \varepsilon(k_c)$. Equation (6) still holds in the two-band case, but the rotation Eq. (5) in spin space is now replaced by a more general unitary matrix operating in a four-dimensional space spanned by $|\sigma m\rangle$ with the spin $\sigma = \uparrow, \downarrow$ and subband index $m = a, b$. The analogue of Eq. (12) in the two-band case is

$$F = T(1-T)\, p\, \left(|R_{\uparrow a,\downarrow a}|^2 + |R_{\uparrow a,\uparrow b}|^2 + |R_{\uparrow a,\downarrow b}|^2\right). \quad (14)$$

We find[6] for the first column of $\mathbf{R}$

$$R_{\uparrow a;\sigma m} = \frac{e^{-i\theta_R/2}}{2}\begin{pmatrix} \cos(\theta_d/2) + e^{i\theta_R} \\ -i\cos(\theta_d/2) + ie^{i\theta_R} \\ -i\sin(\theta_d/2) \\ \sin(\theta_d/2) \end{pmatrix}, \quad (15)$$

where $\theta_d = \theta_R d/k_F$, and $d \equiv \langle a|d/dy|b\rangle$ ("intersubband coupling"). In the absence of intersubband coupling, $\theta_d = 0$, Eq. (15) reduces to the rotation Eq. (5) in spin space. Inserting Eq. (15) into Eq. (14), we obtain

$$f_p = \frac{p}{2}\left(1 - \cos\left(\frac{\theta_d}{2}\right)\cos\theta_R + \frac{1}{2}\sin^2\frac{\theta_d}{2}\right). \quad (16)$$

Again, the above expression reduces to the strictly 1D case Eq. (13) for $\theta_d = 0$. Fig. 3(b) illustrates the effect of the additional spin-rotation θ_d (intersubband coupling) on the reduced Fano factor. This extra modulation arises because impinging electrons with energies at the subband crossing undergo further spin rotation due to channel mixing at k_c, cf. Fig. 2(b). The parameters θ_R and θ_d can in principle be changed independently by changing the width w of the channel since $\theta_d \propto d \propto 1/w$.

To summarize, we find that a local Rashba spin-orbit coupling in a beamsplitter geometry has a strong influence on the current fluctuations (shot noise) for spin-polarized currents. This provides a tool for probing spin polarized current sources and the Rashba coupling strength.

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